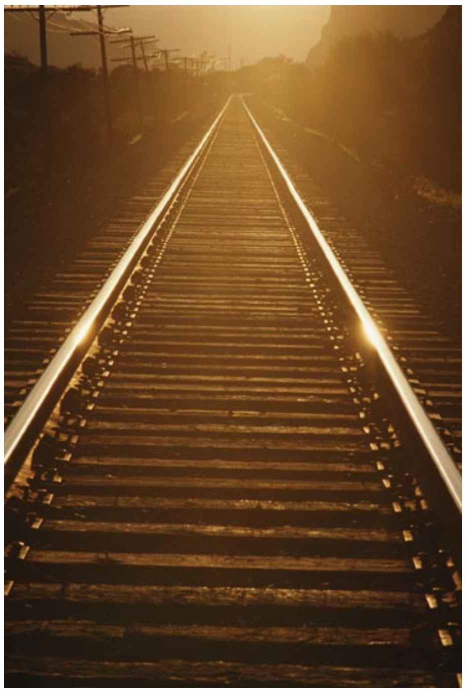
Introduction to Shock & Vibration

- Definitions
- What is Vibration
- Mechanical Parameters
- Mass-spring Systems
- How to Quantify Vibration
- Signal Types
- Time Signal Descriptors
- Conversions:
  Acceleration, Velocity, Displacement
- Units



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Abstract

The lecture gives an introduction to vibration through a description of the most common mechanical parameters leading to the behaviour of simple mass-spring systems. Furthermore the different types of signals and their description is treated and the conversion between the different parameters is described mathematically and graphically. Finally the measurement units are defined.

LECTURE NOTE

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Definitions

Vibration is an oscillation wherein the quantity is a parameter defining the motion of a mechanical system

Oscillation is the variation, usually with time, of the magnitude of a quantity with respect to a specified reference when the magnitude is alternately greater and smaller than the reference

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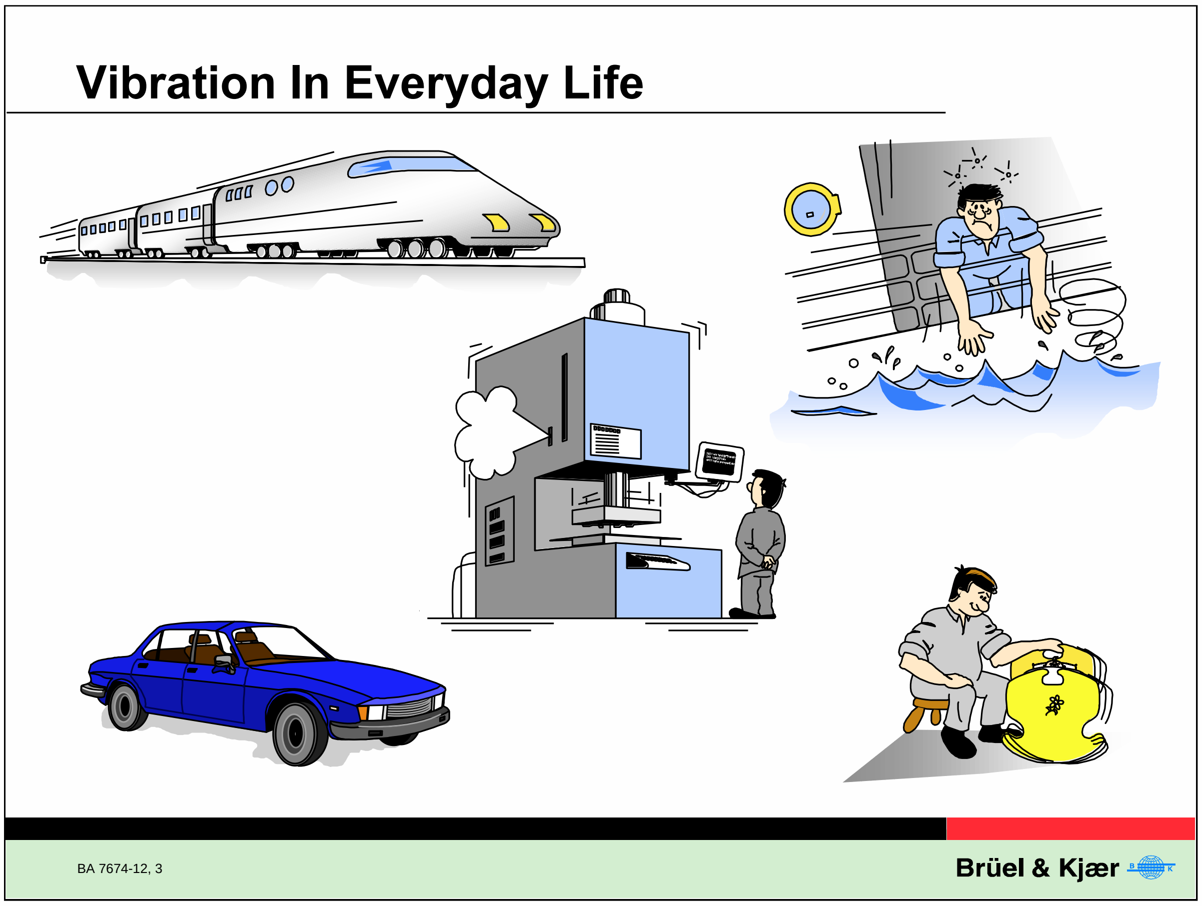
Definitions

It is practical to know more precisely what we are going to talk about. These definitions are adapted from the “Shock and Vibration Handbook” by Harris and Crede (see literature list).

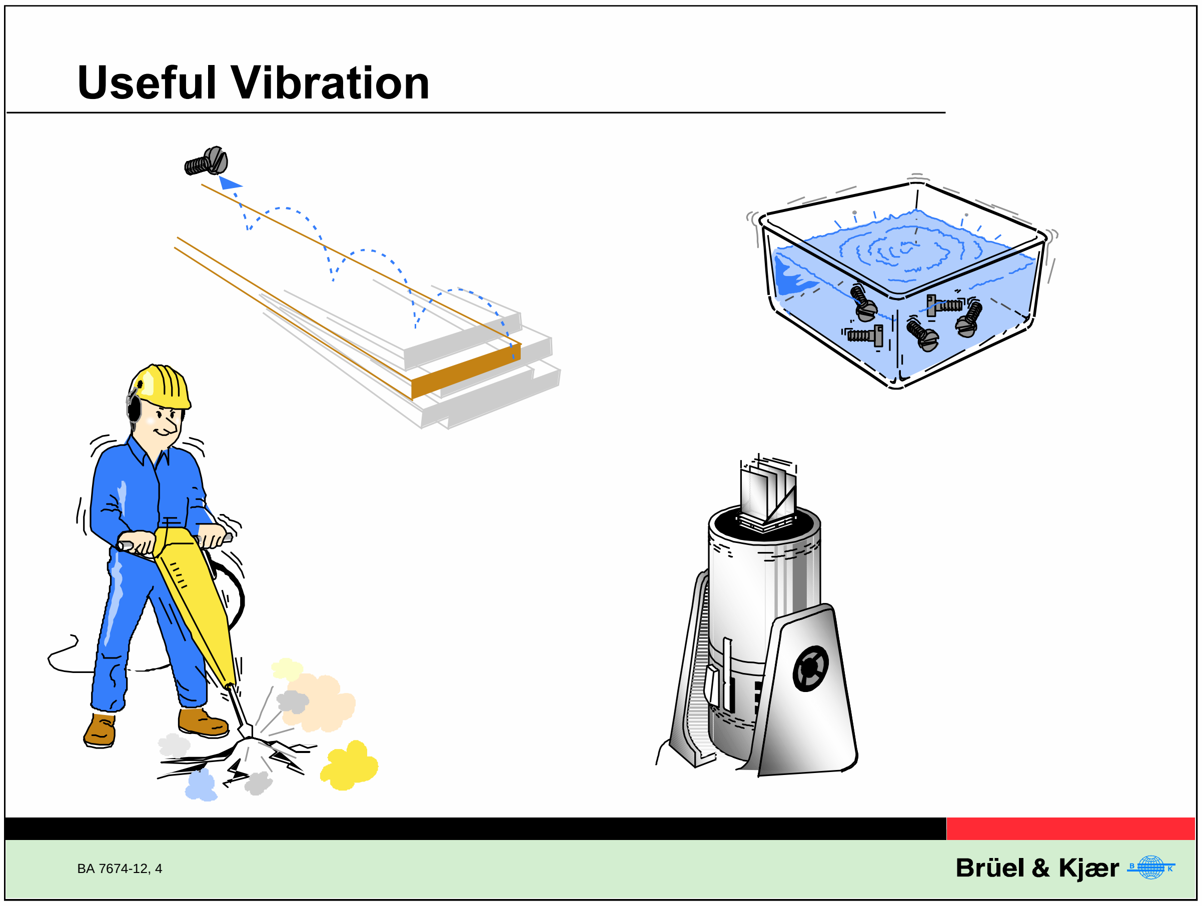


What is Vibration?

Vibration is mechanical oscillation about a reference position. Vibration is an everyday phenomenon, we meet it in our homes, during transport and at work. Vibration is often a destructive and annoying side effect of a useful process, but is sometimes generated intentionally to perform a task.

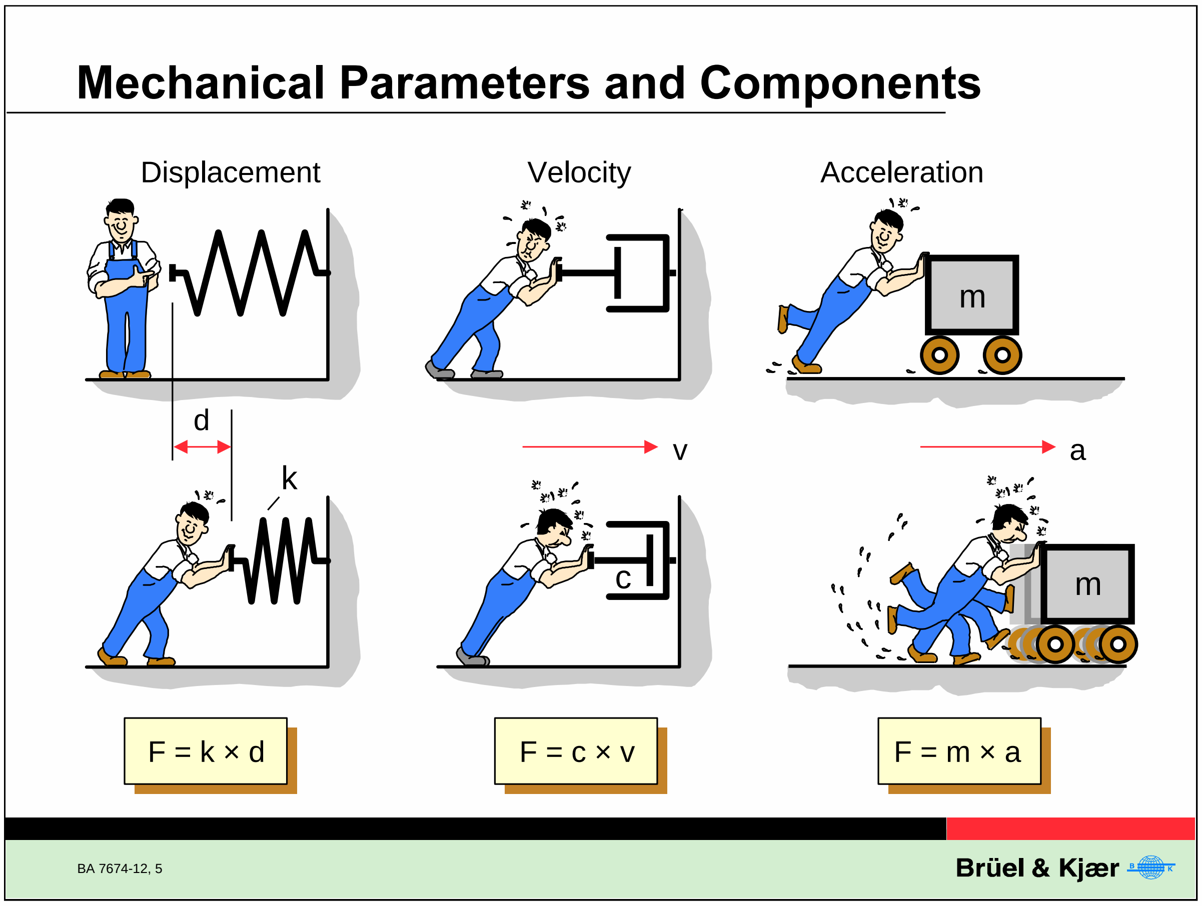
Vibration of machines

Vibration is a result of dynamic forces in machines which have moving parts and in structures which are connected to the machine. Different parts of the machine will vibrate with various frequencies and amplitudes. Vibration causes wear and fatigue. It is often responsible for the ultimate breakdown of the machine.



Useful Application of Vibration

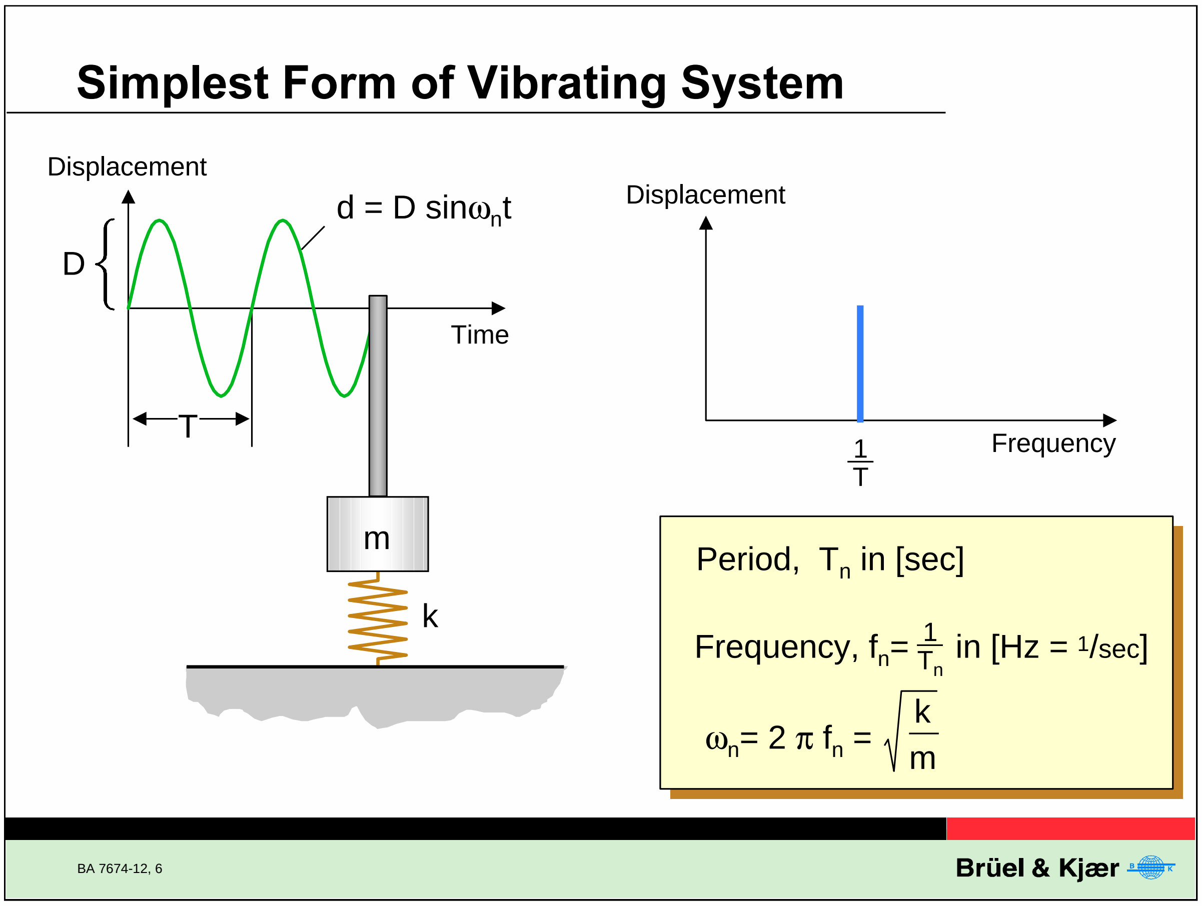
Vibration is generated intentionally in component feeders, concrete compactors, ultrasonic cleaning baths and pile drivers, for example. Vibration testing machines impart vibration to objects in order to test their resistance and function in vibratory environments.



Mechanical Parameters

Before going into a discussion about vibration measurement and analysis, we will examine the basic mechanical parameters and components and how they interact.

All mechanical systems contain the three basic components: spring, damper, and mass. When each of these in turn is exposed to a constant force they react with a constant displacement, a constant velocity and a constant acceleration respectively.



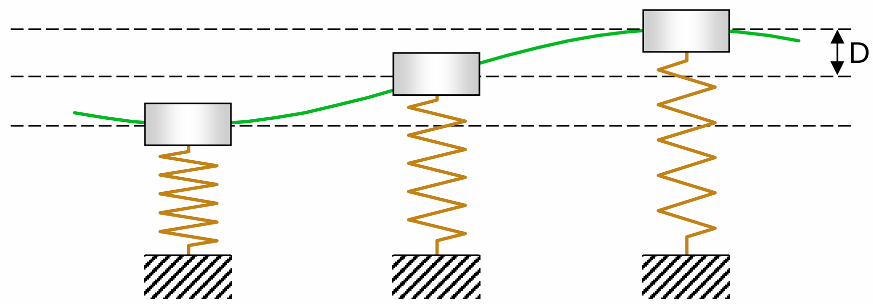
Mass and Spring

Once a (theoretical) system of a mass and a spring is set in motion it will continue this motion with constant frequency and amplitude. The system is said to oscillate with a sinusoidal waveform.

The Sine Curve

The sine curve which emerges when a mass and a spring oscillate can be described by its amplitude (D) and period (T). Frequency is defined as the number of cycles per second and is equal to the reciprocal of the period. By multiplying the frequency by 2π the angular frequency is obtained, which is again proportional to the square root of spring constant k divided by mass m. The frequency of oscillation is called the natural frequency f_n. The whole sine wave can be described by the formula $d = D\sin\omega_n t$, where d = instantaneous displacement and D = peak displacement.

Free Vibration



Energy transfer between Kinetic and Potential Energy
(assuming no damping)

Δ Kinetic Energy = - Δ Potential Energy

$$\tfrac{1}{2} m V^2 = \tfrac{1}{2} k D^2 \text{ , and } V = (2\pi f_n)D$$

$$\tfrac{1}{2} m (2\pi f_n)^2 D^2 = \tfrac{1}{2} k D^2$$

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

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Free undamped vibration

When a free undamped mass-spring system is set into oscillation the added energy is constant, but changes form from kinetic to potential during the motion.

At maximum displacement the velocity and therefore also the kinetic energy is zero, while the potential energy is $1/2kD^2$. At the equilibrium position the potential energy is zero and the kinetic energy is maximum at $1/2mV^2$.

For the sinusoidal motion

$$d = D \sin\omega_n t$$

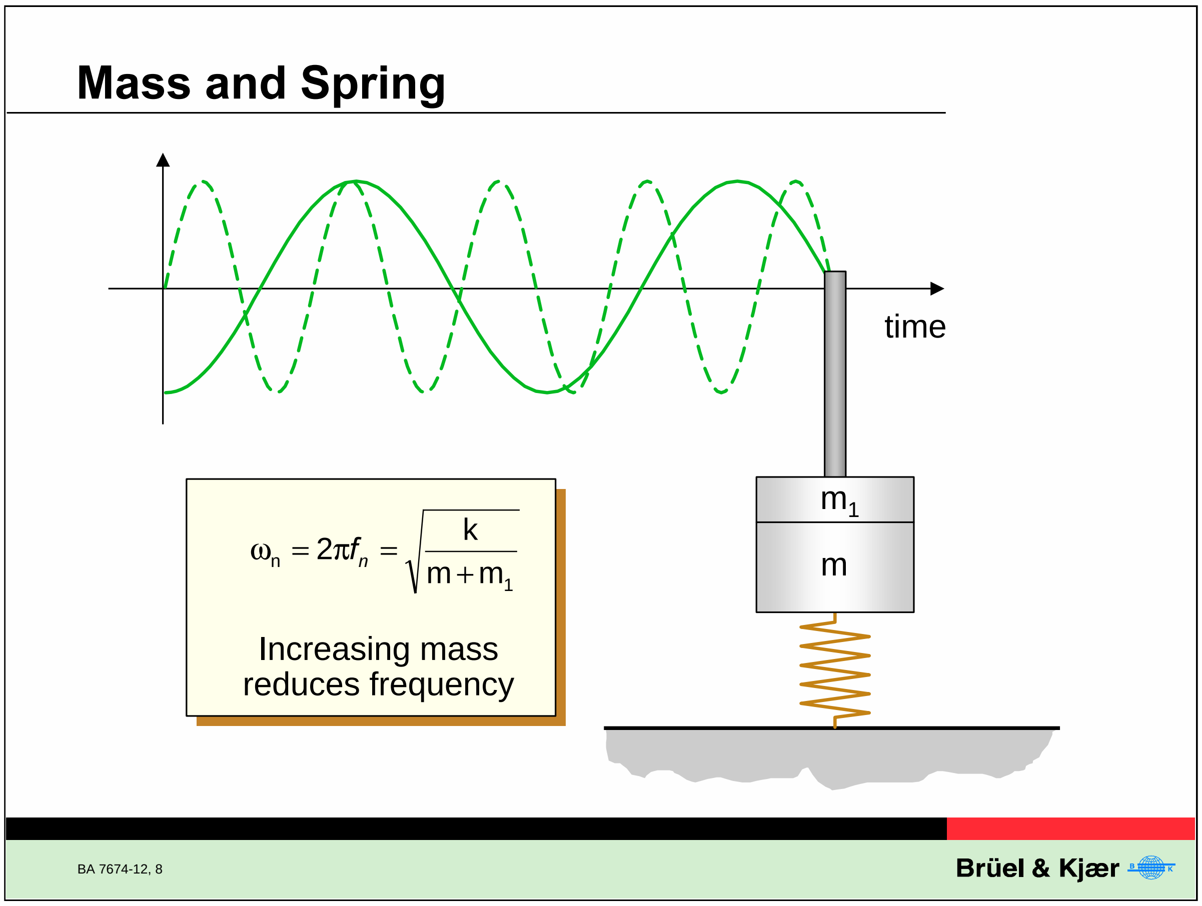
we can also find the velocity by differentiating:

$$v = \frac{d(D\sin\omega_n t)}{dt} = \omega_n D\cos\omega_n t = V\cos\omega_n t$$

and thereby find $V = 2\pi f_n D$.

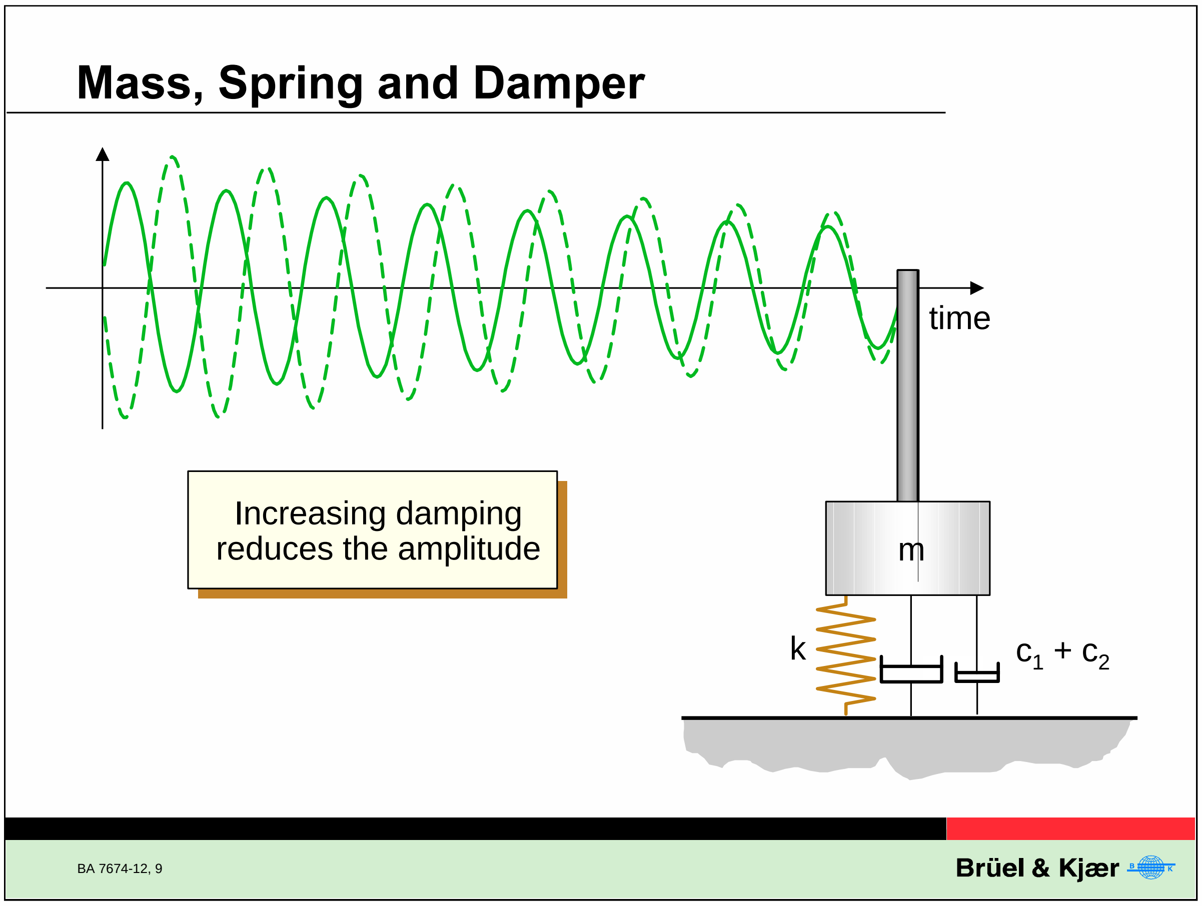
Using energy conservation laws we then get the natural resonance frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$



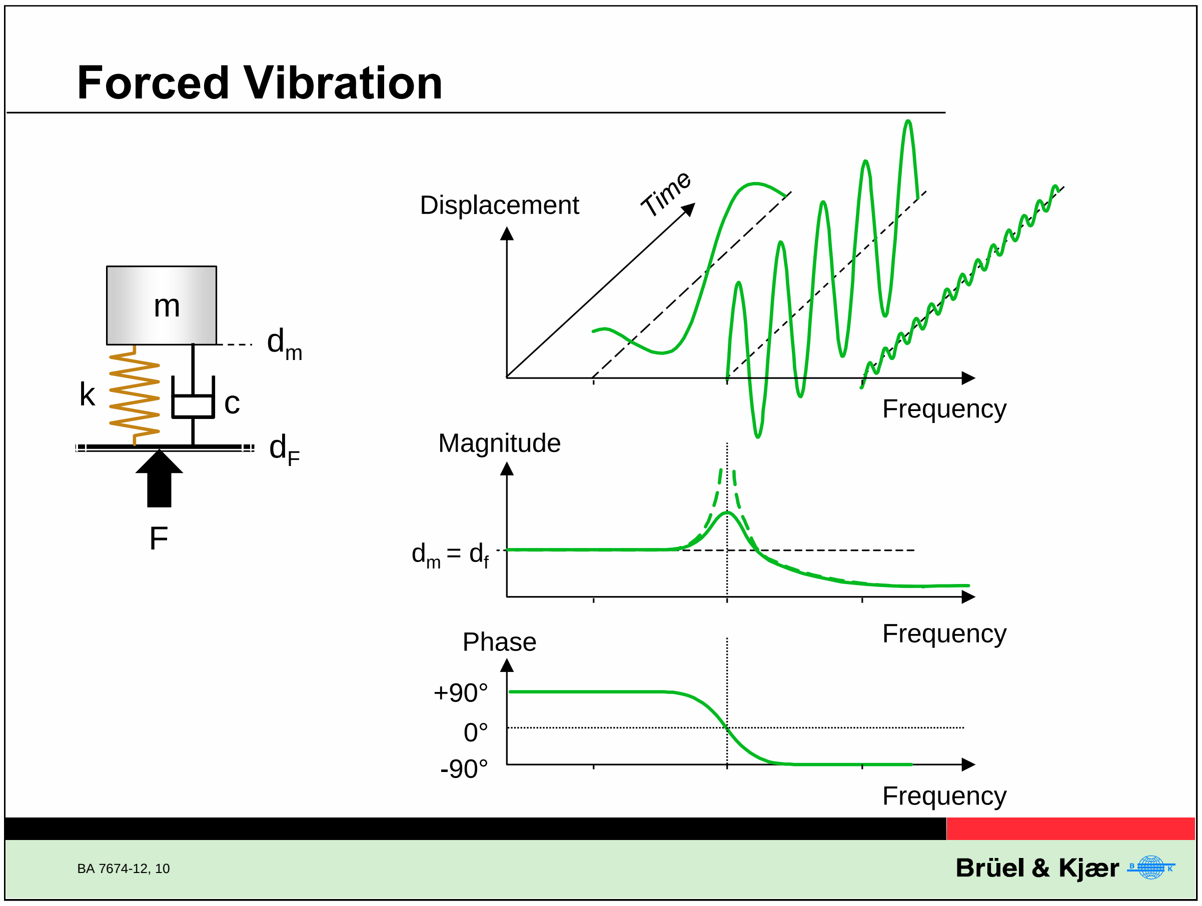
Increase of Mass

An increase in the mass of a vibrating system causes an increase in period i.e. a decrease in frequency.



Mass, Spring and Damper

When a damper is added to the system it results in a decrease in amplitude with time. The frequency of oscillation known as the damped natural frequency is constant and almost the same as the natural frequency. The damped natural frequency decreases slightly for an increase in damping.

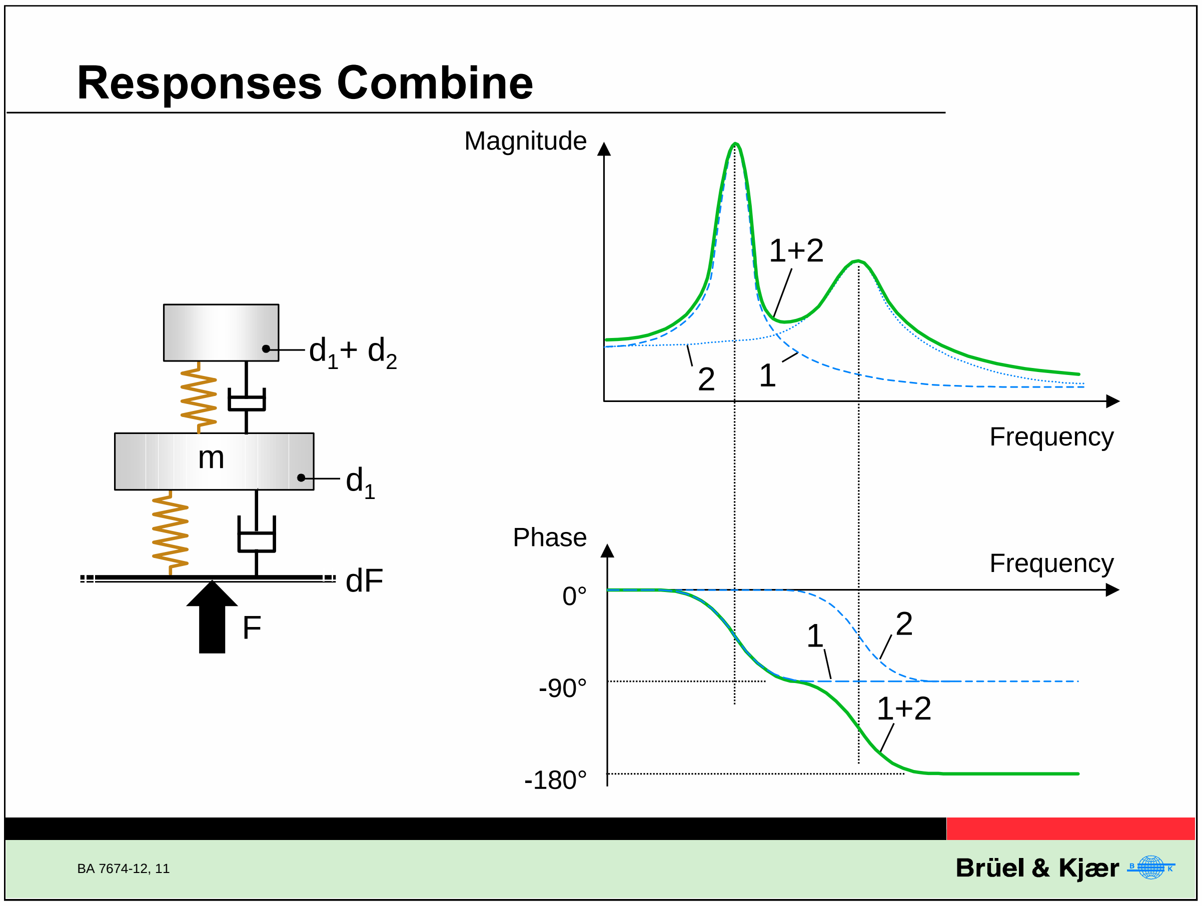


Forced Vibration

If an external sinusoidal force is applied to the system, the system will follow the force, which means that the movement of the system will have the same frequency as the external force. There might, however, be a difference in amplitude (and phase) as shown in the diagram.

For frequencies below its natural frequency, the amplitude of the vibrating system will increase as the frequency is increased, a maximum being reached at the natural frequency. If there was no damping in the system (c = 0), the amplitude would approach infinity.

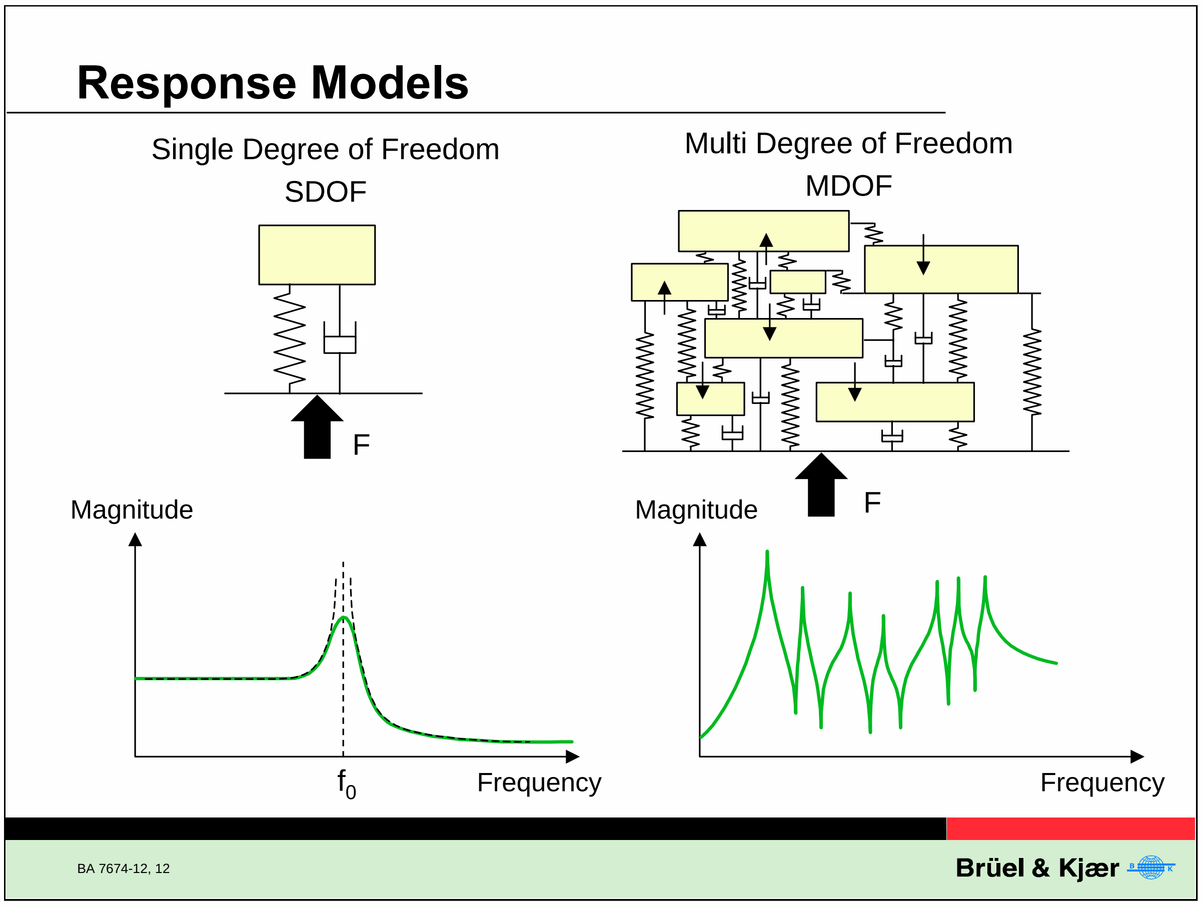
If the frequency of the external force is increased the frequency of the spring/mass/damper system will increase to the same value, but the amplitude (and the phase) will change in accordance with the curves in the diagrams.



Combined responses

When considering real mechanical systems they will normally be more complex than the previous models. A simple example of two masses/springs/dampers is shown here.

In this system we will see the responses combined, and its frequency response function shows two resonance peaks corresponding to the two masse/spring/damper systems.

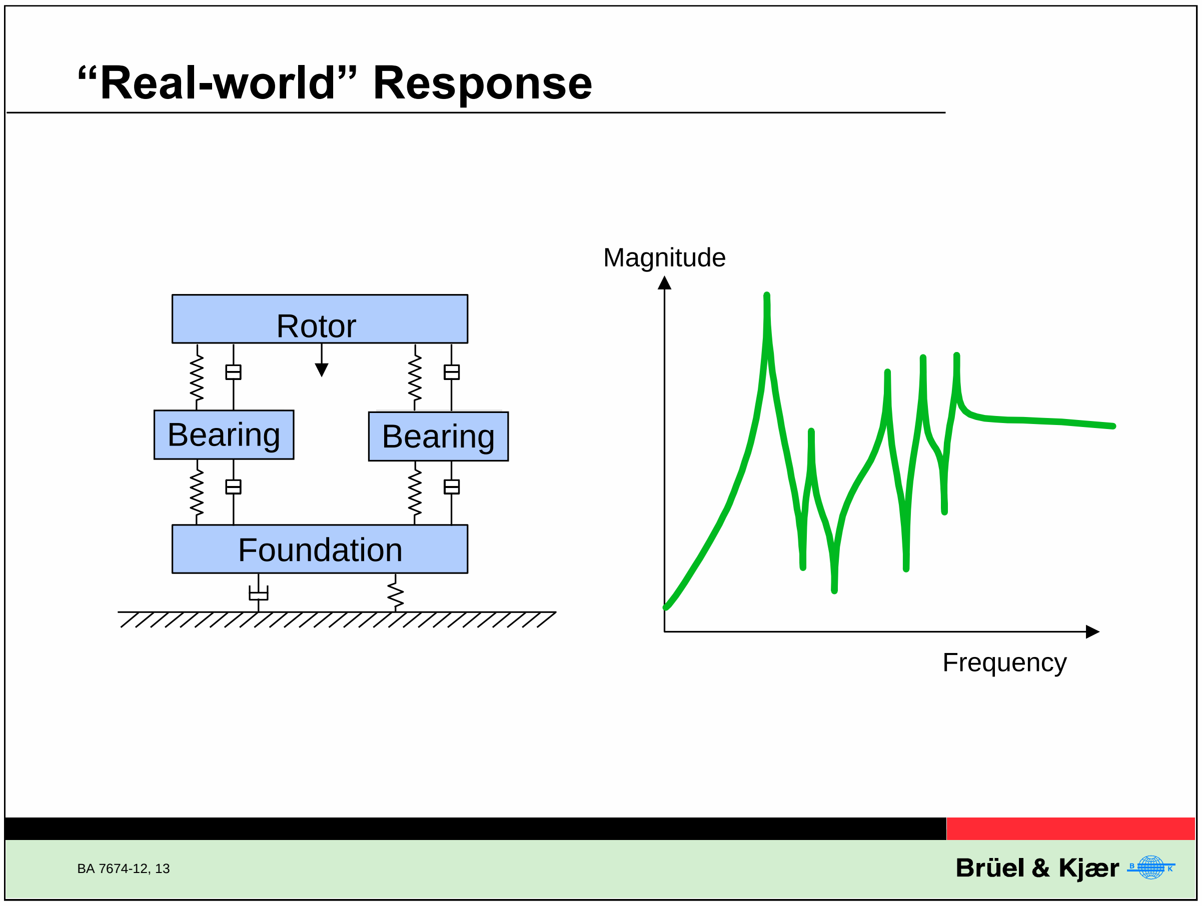


Single Degree of Freedom System

A system consisting of only one mass, one spring and one damper is called a single degree of freedom system (if it can move in one direction only; if this system also can move sideways it is said to have two degrees of freedom and the discussion in the following diagram will apply). The phase is normally ignored in general vibration measurements, but it is very important when system analysis is made.

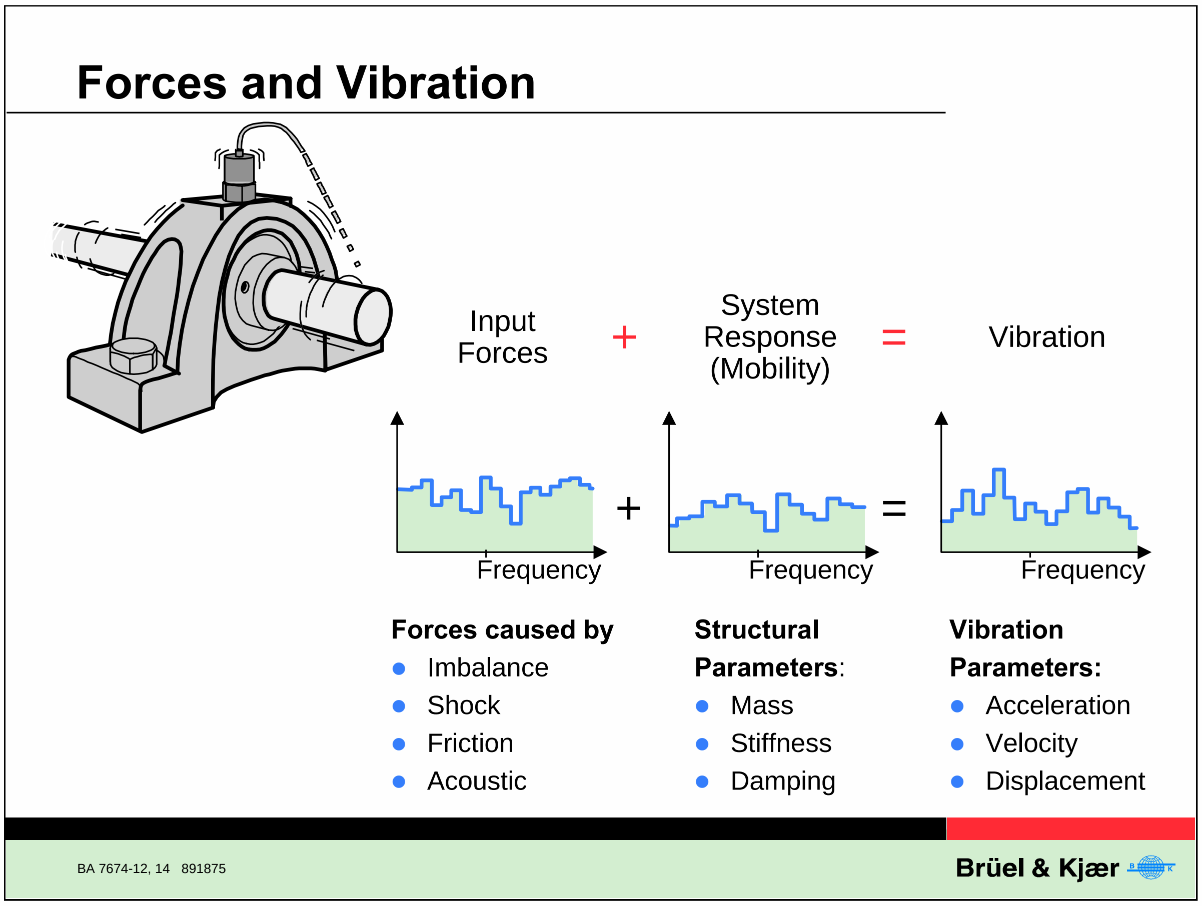
Multi-Degree of Freedom System

If the mechanical system consist of a number of interacting masses, springs and dampers or it can move in more than one direction, it is called a multi-degree of freedom system and the frequency spectrum will have one peak for each degree of freedom. Most systems are multi-degree of freedom systems, although it can often be difficult to separate the different mechanical components and even more difficult to design models as simple as this!



Real-world Response

In most cases even simple systems are to be considered multi degree of freedom systems as illustrated here by a simple rotor in a couple of bearings.

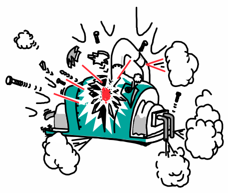


Forces and vibration

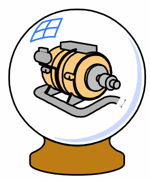
A system will respond to an input force with a certain motion, depending on what we call the mobility of the system. Knowing the force and the mobility permits us to calculate the vibration.

Modal analysis or other methods are used to model systems. Once the model is created we can calculate its mobility for a force input at a certain point, and thereby predict vibration at different locations. Such models can also in some cases be used to calculate the load on the structure to predict failure.

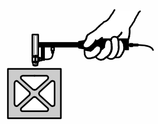
Why Do We Measure Vibration?



- To verify that frequencies and amplitudes do not exceed the material limits (e.g. as described by the Wöhler curves)
- To avoid excitation of resonances in certain parts of a machine



- To be able to dampen or isolate vibration sources
- To make conditional maintenance on machines



- To construct or verify computer models of structures (system analysis)

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Why measure vibration?

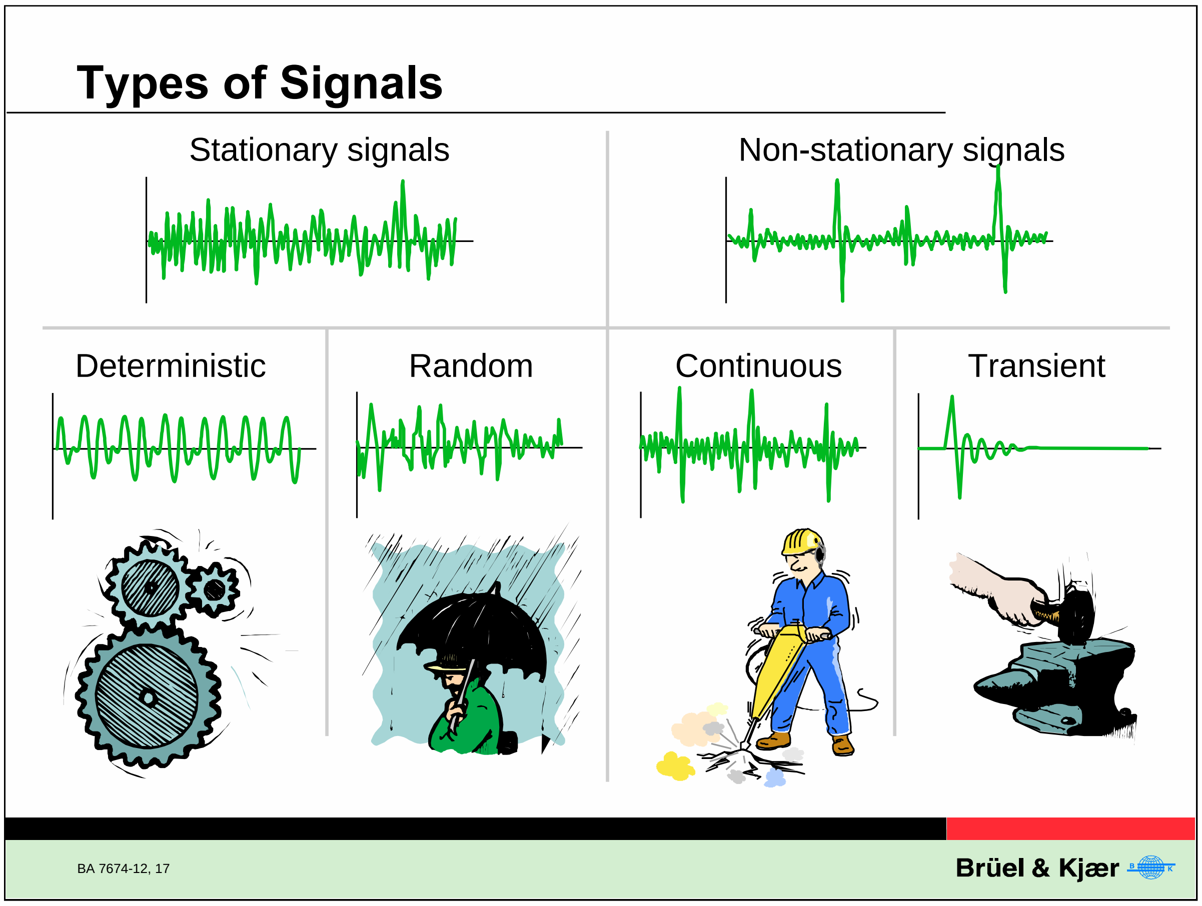
A number of reasons are listed here.

The Wöhler curve is a curve describing the stress level up to which a structure can be loaded a certain number of times (endurance strength). At high stresses the load can only be carried a few times, but reducing the stress increases the number of cycles to failure. For most metals there exists an endurance limit for which the endurance becomes infinite. This stress level is very important, and it is often found by subjecting the object to 10.000.000 cycles of stress, based on the experience that this number is sufficient to reach the endurance limit.

How do We Quantify Vibration?

- We make a measurement
- We analyse the results (levels and frequencies)

In order to make the analysis, we must first talk about the types of vibration signals we might encounter and how we measure these signals



Signals

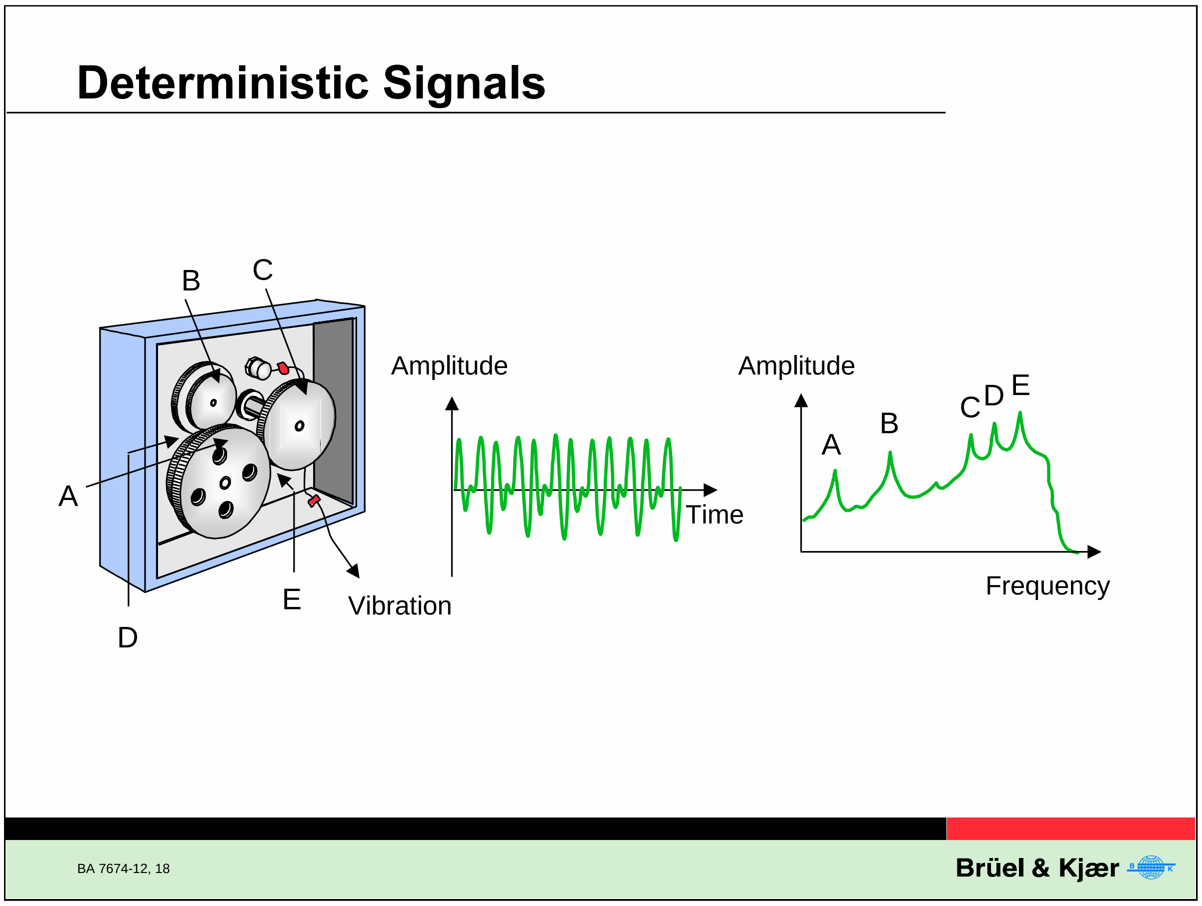
Basically a distinction between Stationary Signals and Non-stationary Signals has to be made. Stationary Signals can again be divided into Deterministic Signals and Random Signals, and Non-stationary Signals into Continuous and Transient signals.

Stationary deterministic signals are made up entirely of sinusoidal components at discrete frequencies.

Random signals are characterised by being signals where the instantaneous value cannot be predicted, but where the values can be characterised by a certain probability density function i.e. we can measure its average value. Random signals have a frequency spectrum which is continuously distributed with frequency.

The continuous non-stationary signal has some similarities with both transient and stationary signals. During analysis continuous non-stationary signals should normally be treated as random signals or separated into the individual transient and treated as transients.

Transient signals are defined as signals which commence and finish at a constant level, normally zero, within the analysis time.

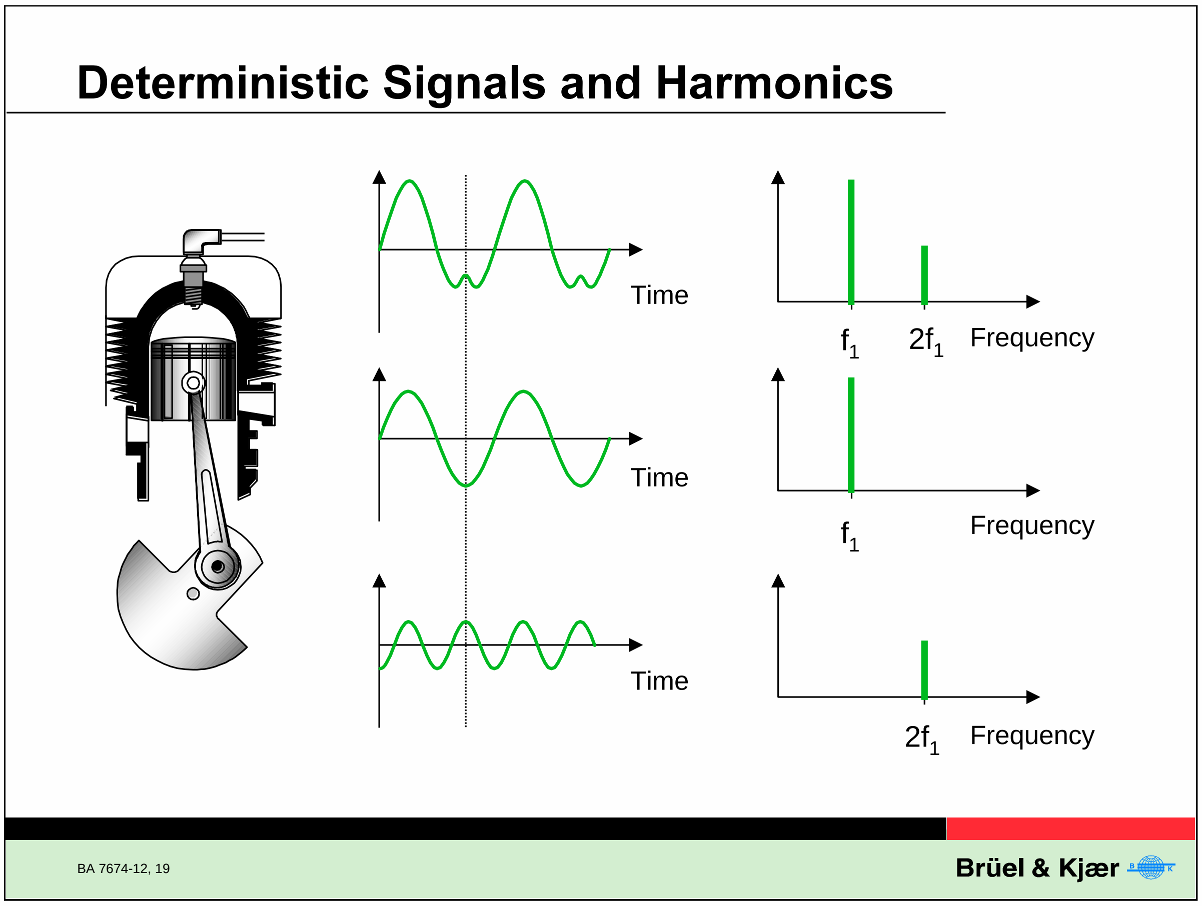


Deterministic Signals

The vibration signal from a gearbox could look like the one shown here. In the frequency domain this signal will give rise to a number of separate peaks (discrete frequency components) which through knowledge of the number of teeth on the gearwheels and their speed can be related back to particular parts of the system. The signal here is called deterministic, since the instantaneous value of the signal is predictable at all points in time.

The role of frequency analysis

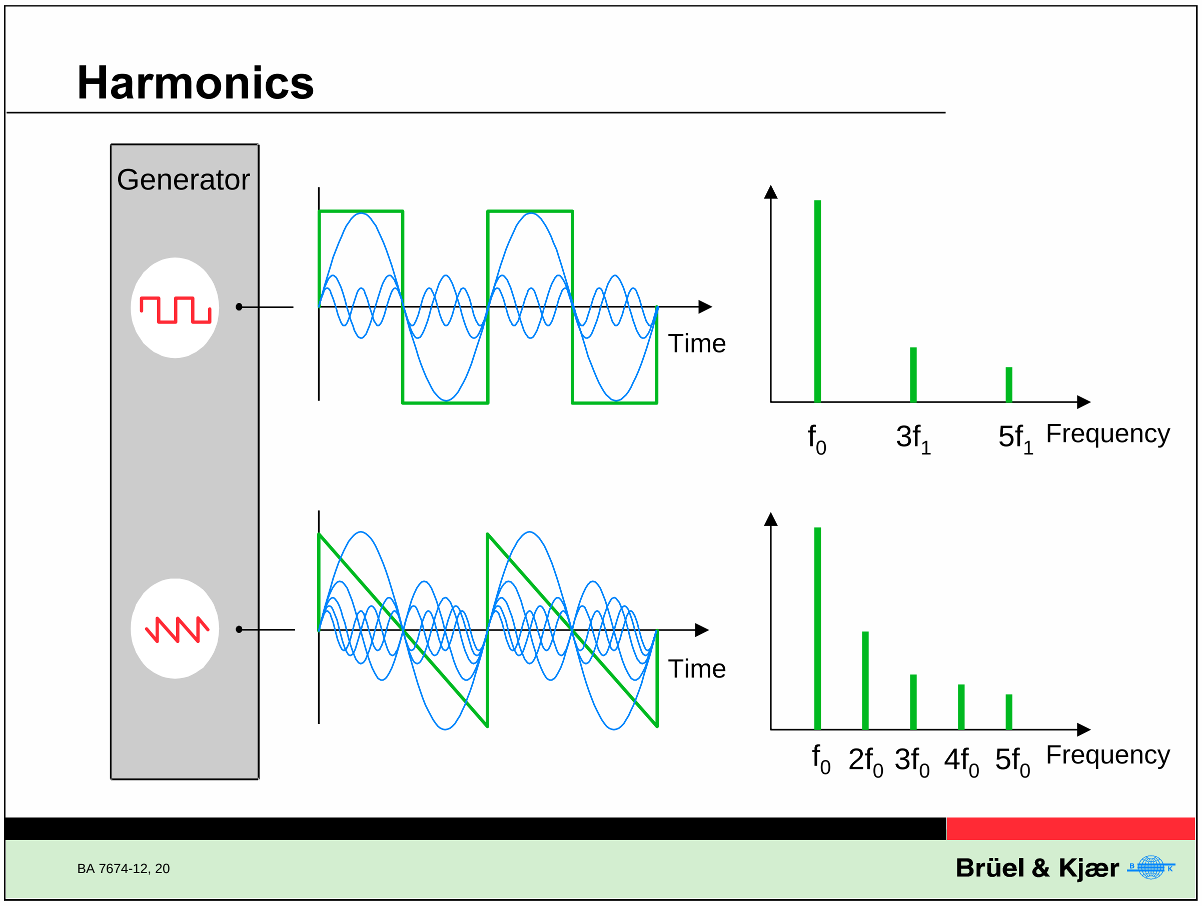
The frequency spectrum gives in many cases a detailed information about the signal sources which cannot be obtained from the time signal. The example shows measurement and frequency analysis of the vibration signal measured on a gearbox. The frequency spectrum gives information on the vibration level caused by rotating parts and tooth meshing. It hereby becomes a valuable aid in locating sources of increased (undesirable) vibration from these and other sources.



Vibration Signals

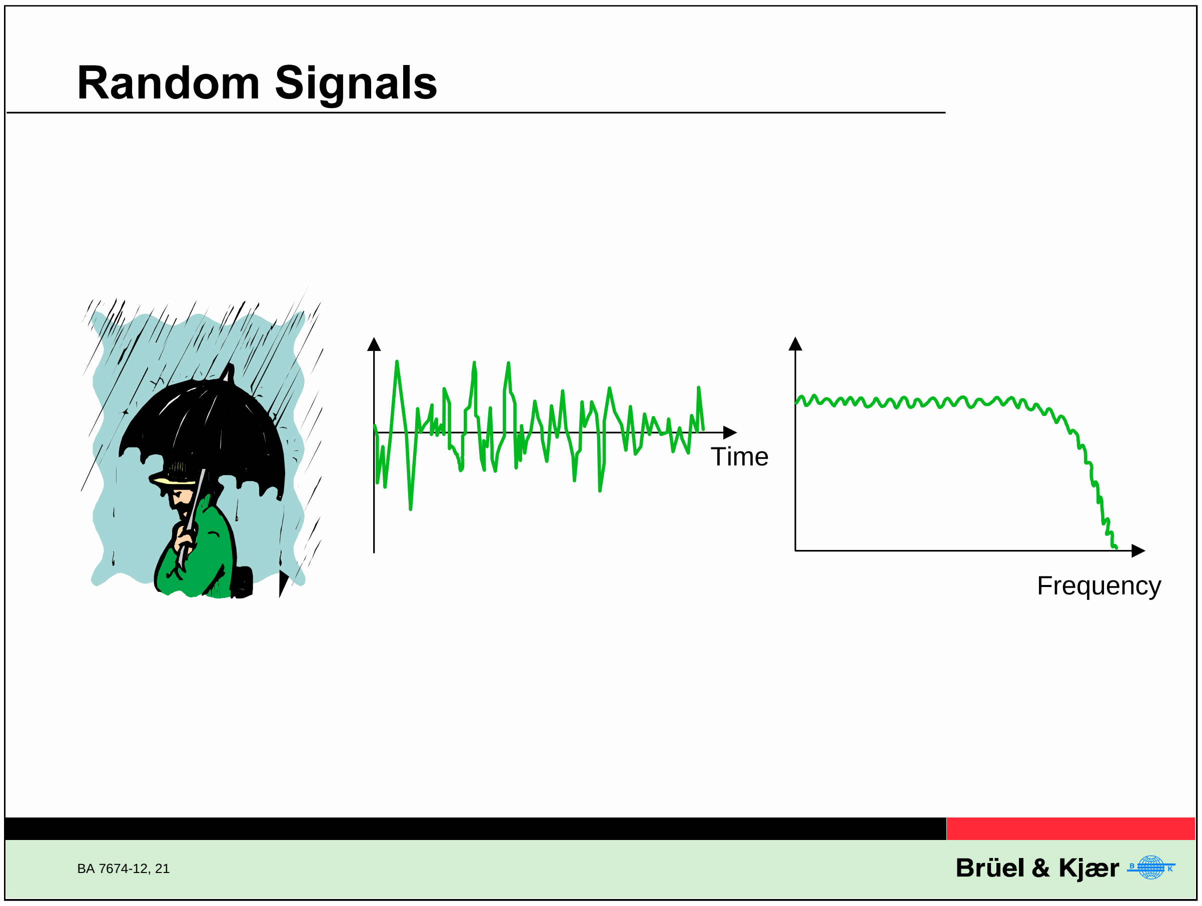
The motion of a mechanical system can consist of a single component at a single frequency as with the system described in one of the previous examples; (a tuning fork is another example) or it can consist of several components occurring at different frequencies simultaneously, as for example with the piston motion of an internal combustion engine.

The motion signal is here split up into its separate components both in the time domain and in the frequency domain.



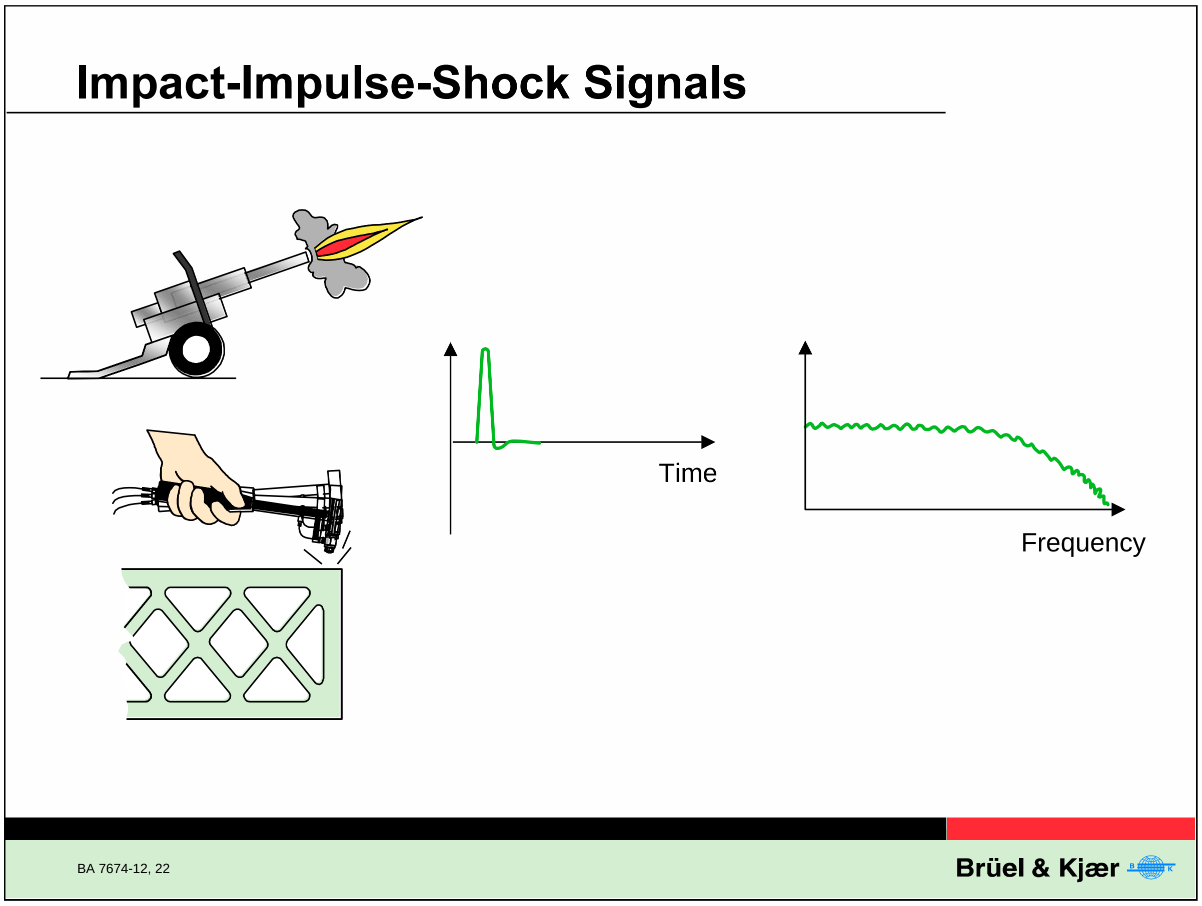
Harmonics

Many non-sinusoidal signals can be separated into a number of harmonically related sinusoids. Two examples are given. The harmonic components are always referred to the fundamental frequency to which they are related.



Random Signals

A typical example of random vibration is that caused by fluid flow. Random signals have no periodic and harmonically related components. They are characterised by totally random movements, so that their instantaneous value cannot be predicted. Random vibration can, however, be described by its statistical properties. Stationary random signals have a frequency spectrum which is no longer concentrated at discrete frequencies, but distributed continuously with frequency.



Shock

Mechanical shock is a short burst of vibratory energy. If the shock is infinitely short it will also have a frequency spectrum which is distributed continuously with frequency. Since a shock will always have a finite length its frequency spectrum will be limited to a band of frequencies.

Which Time Signal Parameter Do We Use

- Peak
- Peak-Peak
- Average
- RMS
- Crest Factor
- Periodicity/Repetition Rate
- Duration

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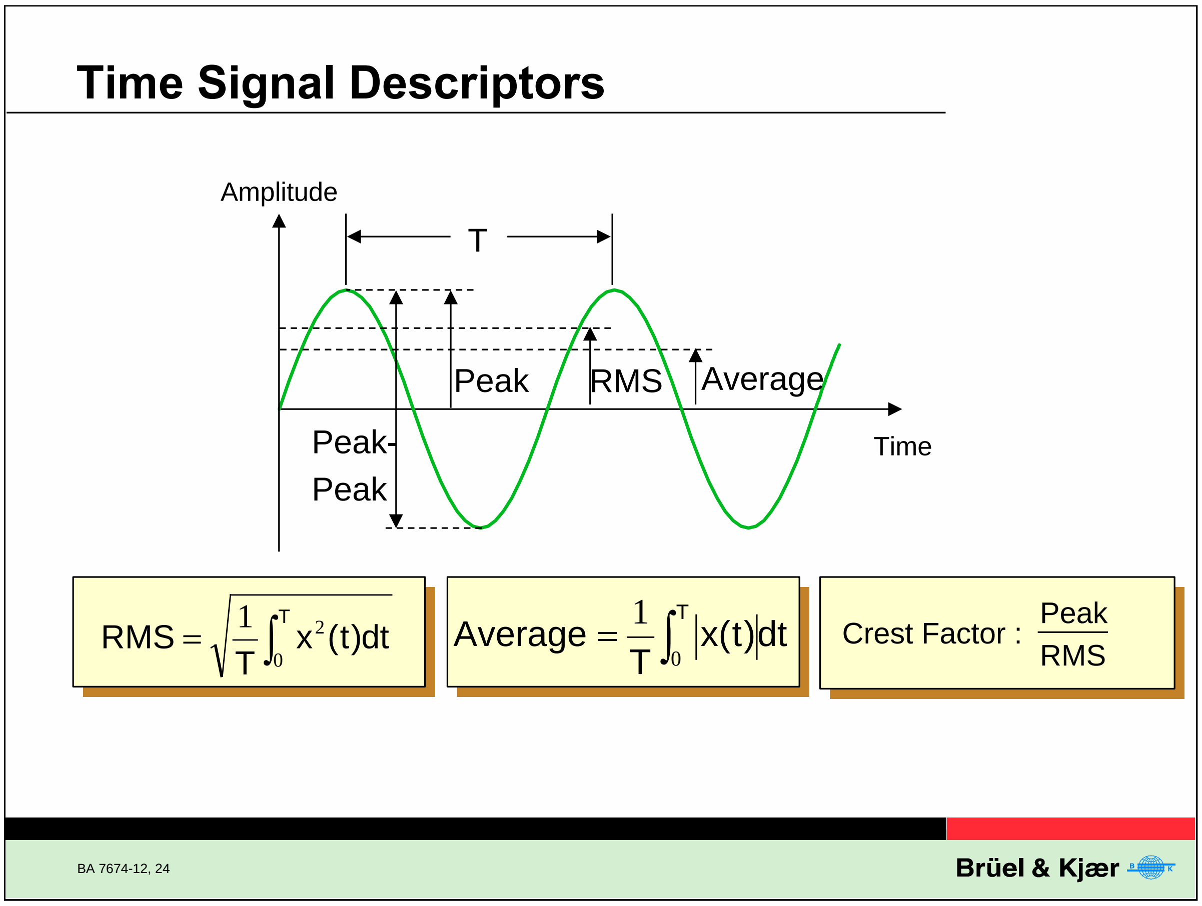
Which parameter to measure?

Measuring on the time signal is the simplest form of analysis.

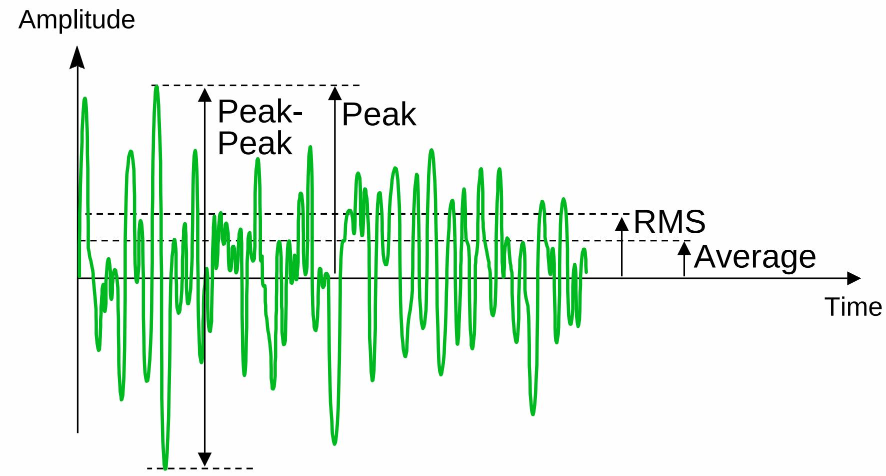
A number of different possibilities are mentioned here.



Signal Level Descriptors

The level of vibration signal can be described in different ways. Peak and peak-to-peak values are often used to describe the level of a vibration signal since they indicate the maximum excursion from equilibrium position. The RMS (Root Mean Square) level is a very good descriptor, since it is a measure of the energy content of the vibration signal.

Time Signal Descriptors



$$\text{RMS} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$$

$$\text{Average} = \frac{1}{T}\int_0^T |x(t)|dt$$

$$\text{Crest Factor :} \ \frac{\text{Peak}}{\text{RMS}}$$

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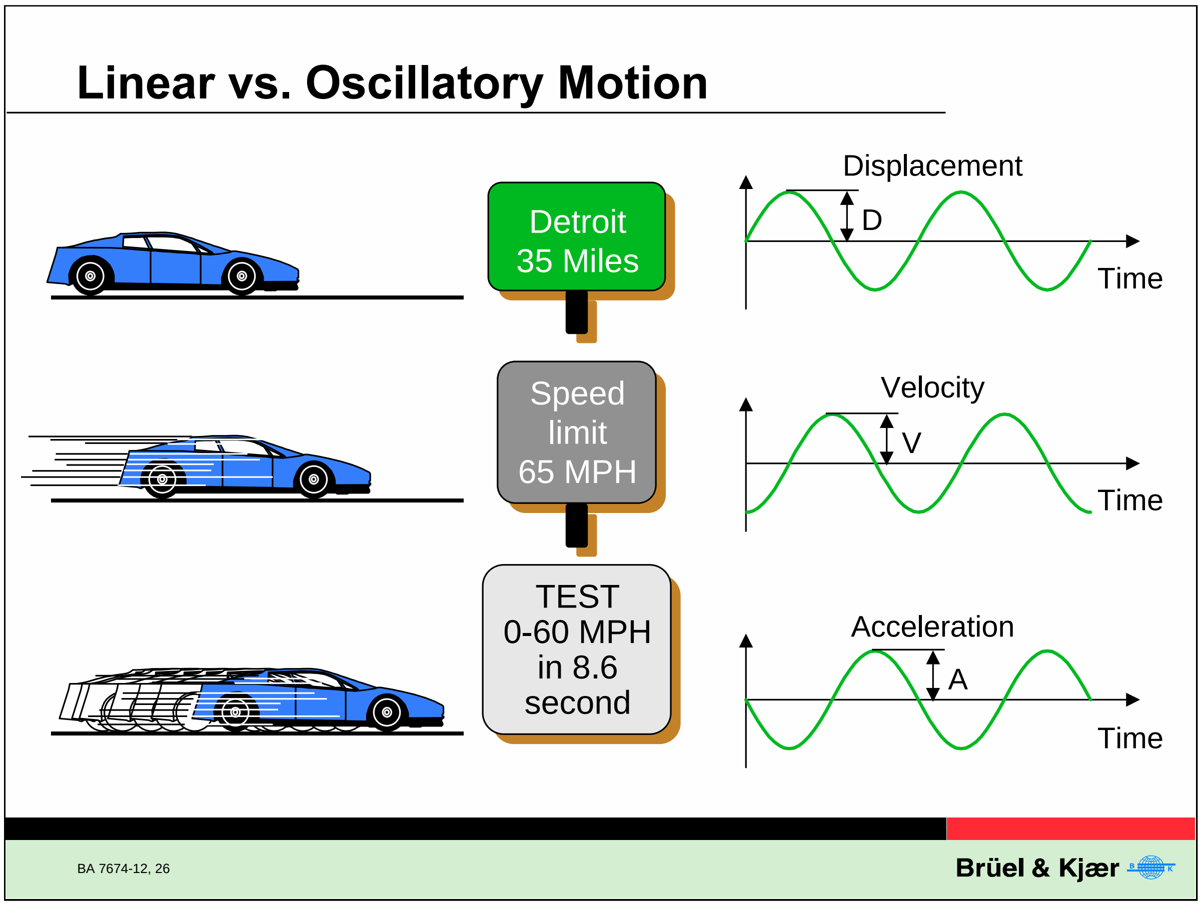
Time Signal Descriptors

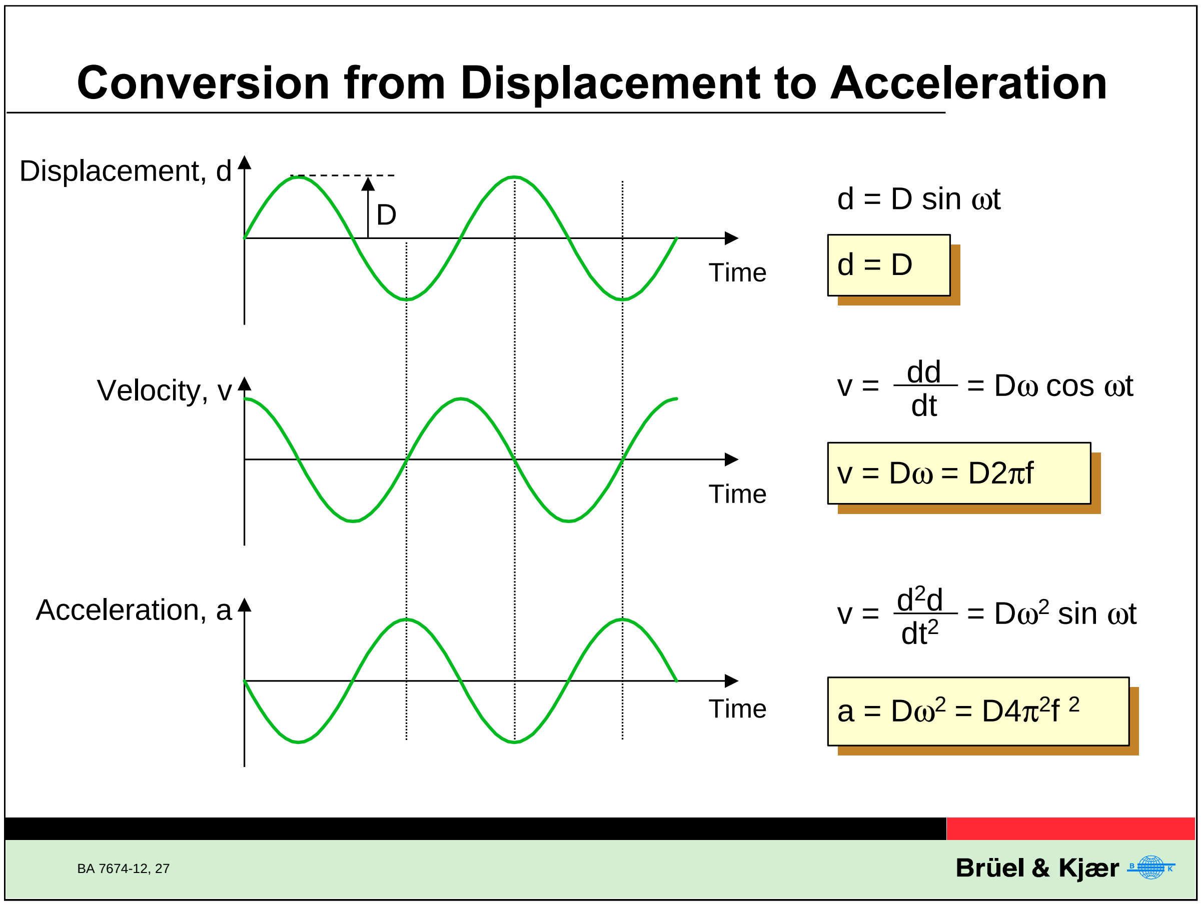
These descriptors are not only used in conjunction with a single sinusoidal signal but also with normal machine vibration signals which are composed of many sinusoidal vibration components.



Linear vs. oscillatory motion

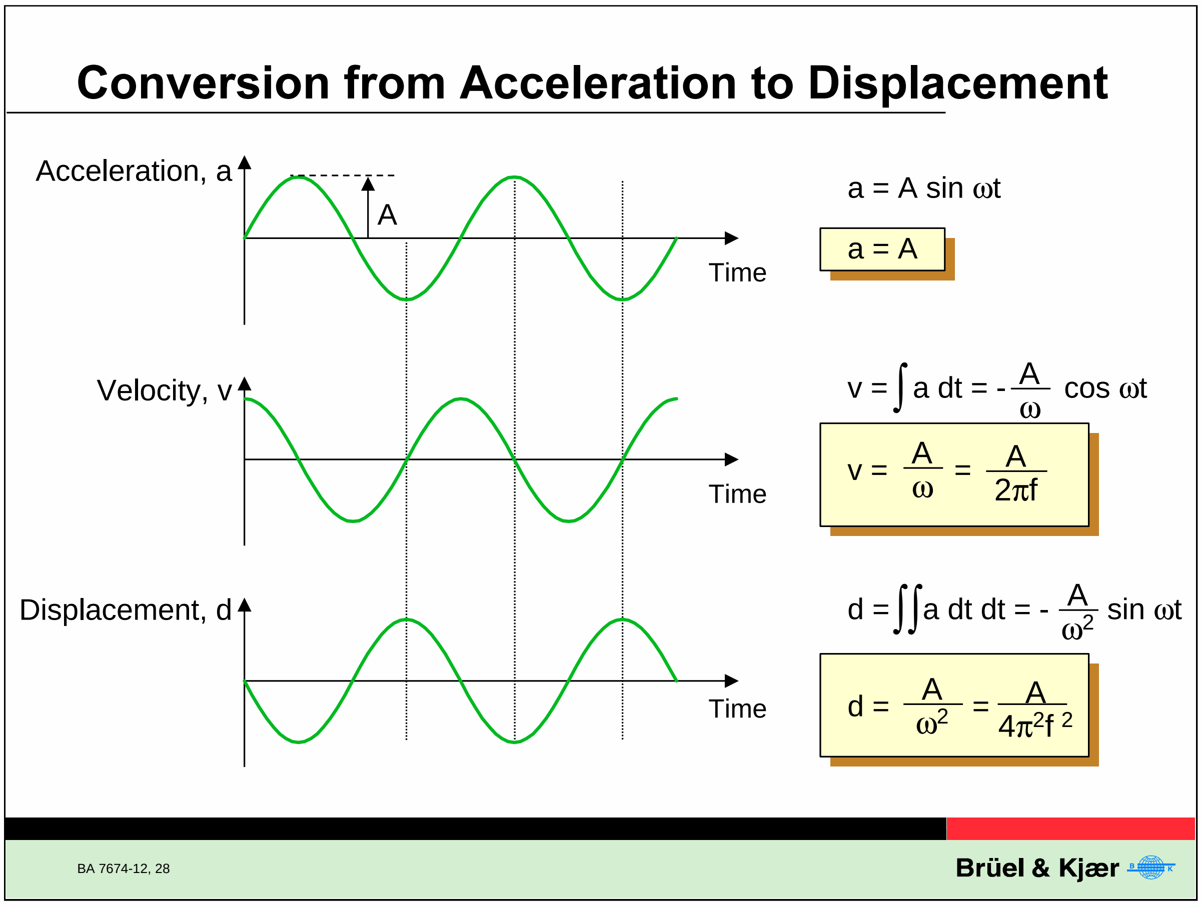
The motion of an object along a straight line can be described in the form of the instantaneous position, its velocity at the given time and its acceleration at that time.

For an oscillatory motion at a certain frequency the three parameters are strictly linked together.



Relationship between d, v and a

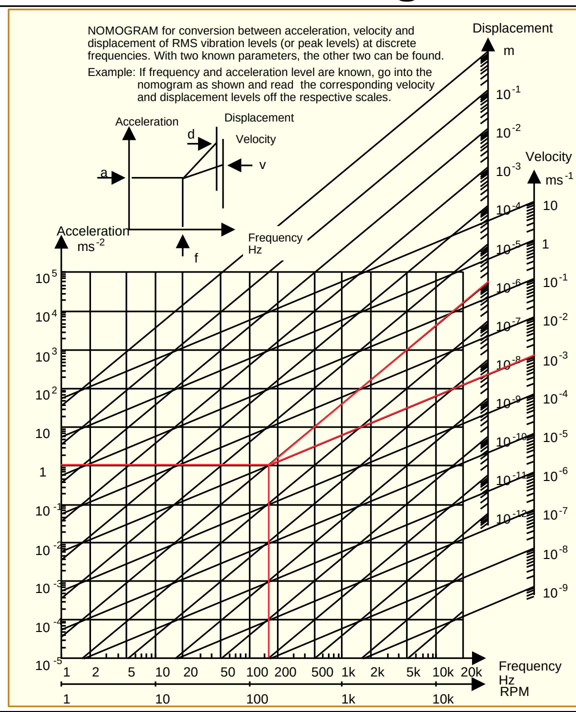
The three mechanical parameters displacement, velocity and acceleration are closely related. If a vibration signal containing only one frequency is considered the form and period of the signal remains the same whether it is the displacement, velocity or acceleration being considered, the main difference is that there is a phase difference between the amplitude-time curves of the three parameters. Knowing the displacement signal, the others can be found through a single and double differentiation of this signal. If the phase difference is ignored (as is normally the case) the numerical values of velocity and acceleration can be found by simple multiplication as shown.

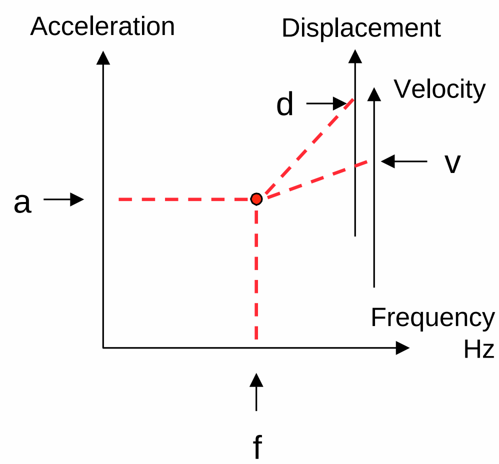


Relationship between a, v and d

If the parameter measured is the acceleration, the two other parameters can just as easily be found through a single and double integration of the acceleration signal. Since integration is easier than differentiation by electronic methods, the best parameter to pick up is acceleration. The electronic equipment takes care of the rest. There are other reasons for choosing acceleration as the best parameter to pick up. This will be explained in following lectures.

Conversion using a Nomogram





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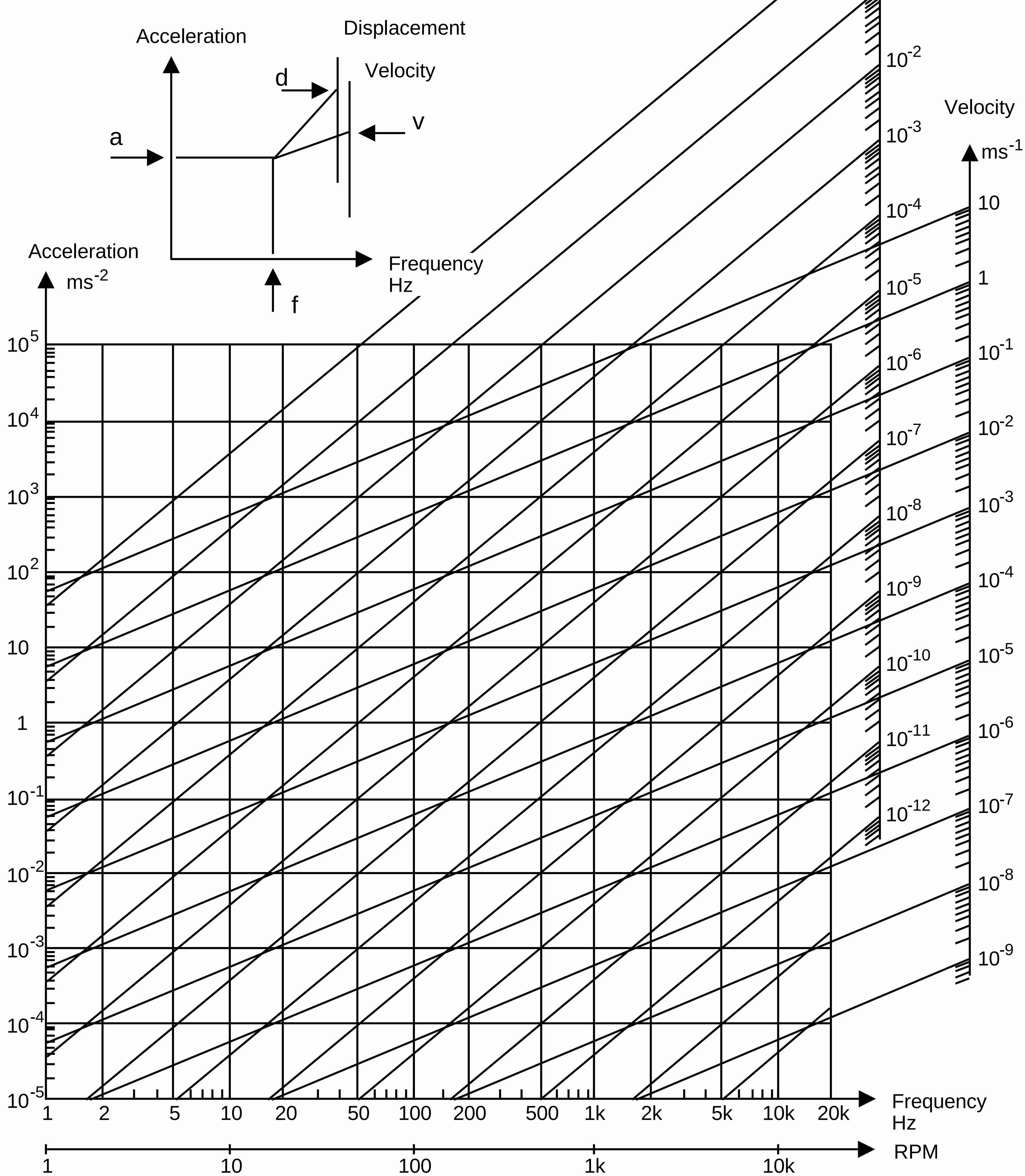
Use of a Nomogram for Conversion between a, v and d

Knowing the frequency of vibration and the acceleration level, the velocity and displacement levels can easily be found by the use of a nomogram. Note that the nomogram is only valid for vibration with a single frequency component and not for a signal containing several frequency components. The details in the nomogram pictured on the transparency cannot be seen very clearly. Therefore the next notes page contains a full size copy for practical use.

NOMOGRAM for conversion between acceleration, velocity and displacement of RMS vibration levels (or peak levels) at discrete frequencies.
With two known parameters, the other two can be found.

Example: If frequency and acceleration level are known, go into the nomogram as shown and read the corresponding velocity and displacement levels off the respective scales.



Units of Vibration Signals

Acceleration a	$1 ms^{-2}$ (m/s^2)	$= 0.102g = 39.4\ in/s^2$
Velocity v	$1 ms^{-1}$ (m/s)	= 3.6 km/h = 39.4 in/s
Displacement d	1m	= 1000 mm = 39.4 in

$$1g \equiv 9.80665\ ms^{-2}$$

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Units

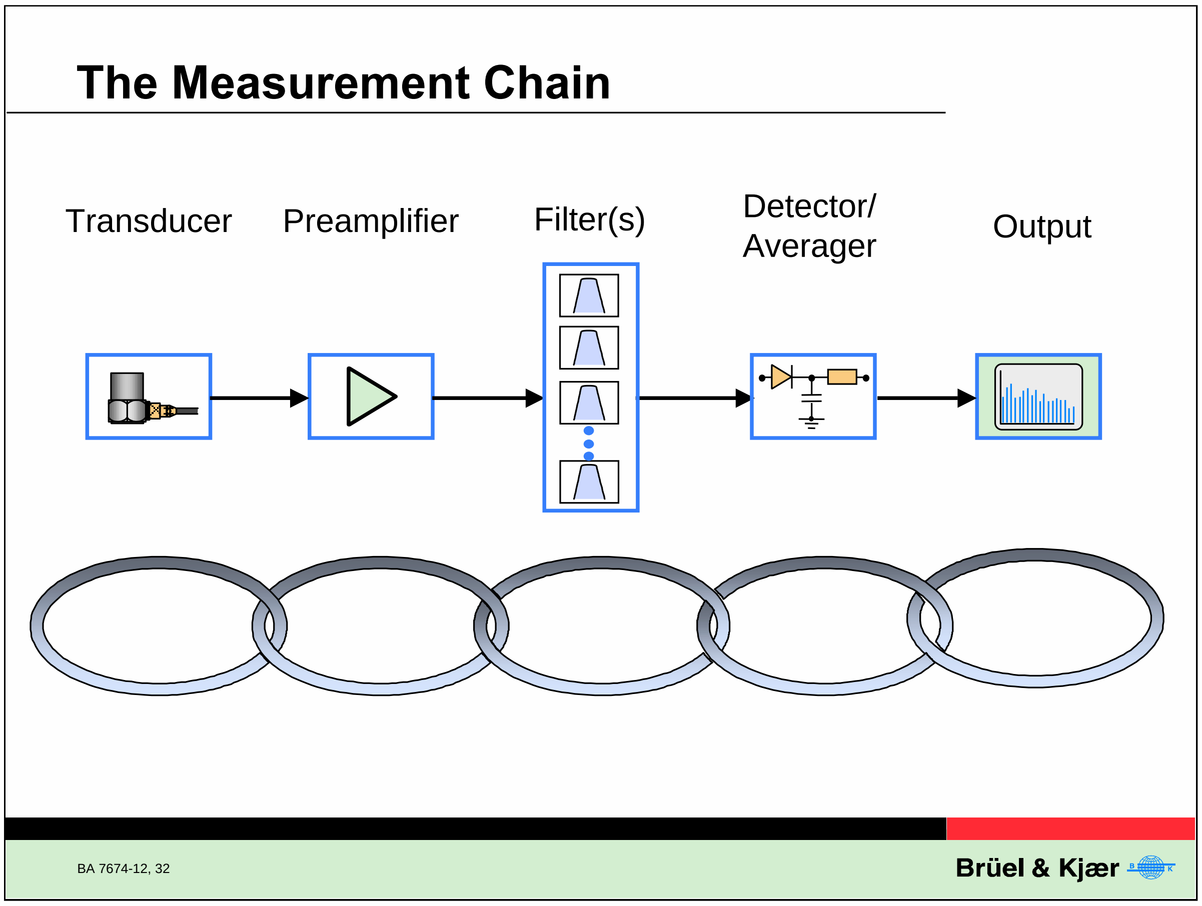
The SI units for acceleration, velocity and displacement are ms^{-2}, ms^{-1} and m. Other units have earlier been used, but in the interests of international standardisation these should be avoided as much as possible.

The unit g often used in vibration work comes from the gravitational acceleration, but is today purely a unit defined as:

$$1g \equiv 9.80665\ ms^{-2}$$

likewise

$$1\ \text{inch} \equiv 25.4\ \text{mm}$$

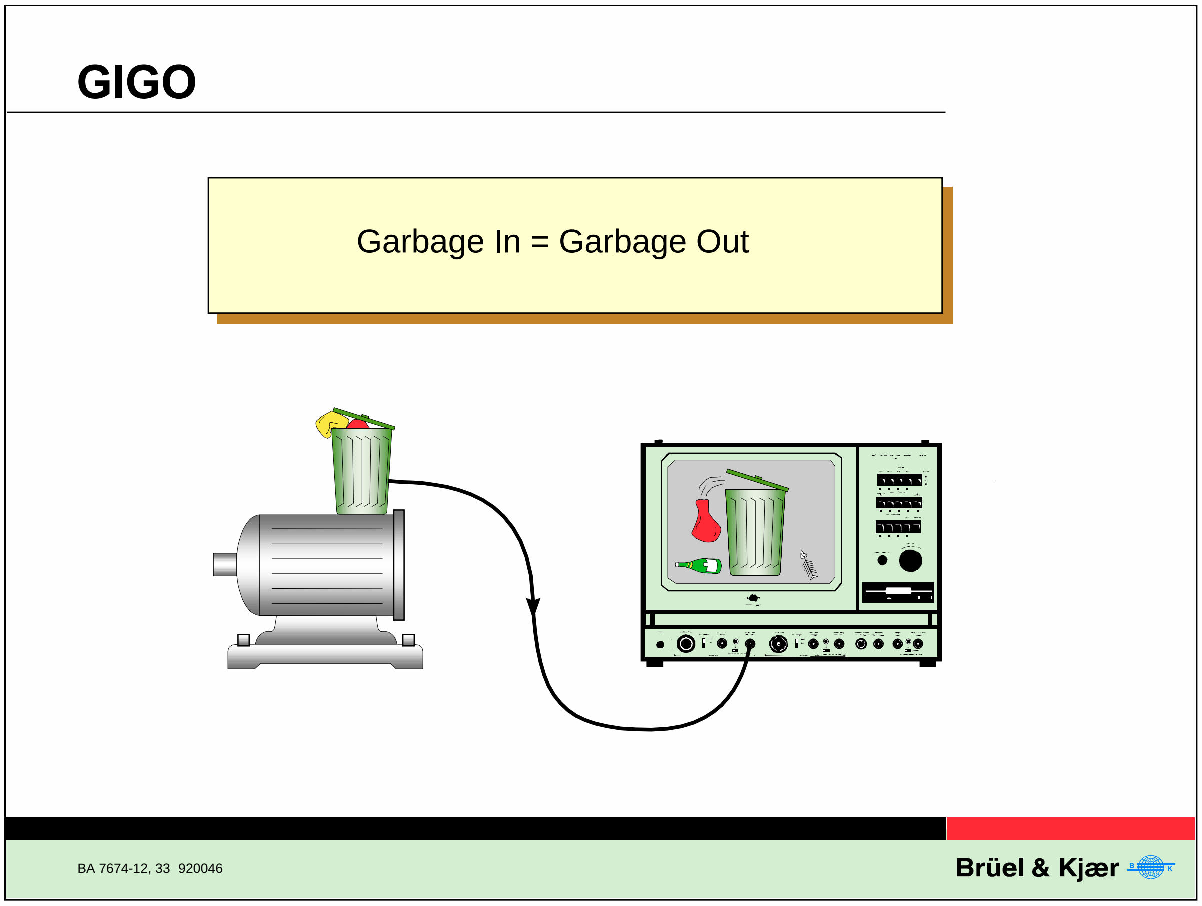


The measurement chain

Now understanding the fundamentals of vibration we need to look at the system or measurement chain to be used for its measurement.

Normally a chain like the one illustrated is used. It contains a proper transducer, preamplifier suitable for the transducer, an analysis system which can be as simple as a mean value detector and as complicated as an FFT analyser and finally an output in the form of a screen picture, a printout or data stored in a computer or on a diskette.

The important thing to remember is that this chain, just as the normal one shown, is no stronger than its weakest member!



Garbage In, Garbage Out.

In development of new cars, monotoring of machines and many other applications of vibration measurement it is of paramount importance to have a very high reliability in the quality of measurement.

Unfortunately, choosing the accelerometer as the front end doesn't automatically give the enhanced reliability suggested. Firstly, the correct type must be chosen, and then it must be used properly as we will see in later lectures.

If the accelerometer used gives bad data for any reason, the entire machine condition monitoring system collapses or we get an unusable new car construction.

A system with Garbage going In, can only give Garbage coming Out. That's the GIGO principle.

Don't forget the concept of the system must also include not only the software and hardware, but also the resources tied up, both human and financial.

Conclusion

You should now have a good understanding of:

- The fundamental nature of vibration
- The mechanical parameters involved
- The types of signals encountered
- The relationships between a, v and d
- The units of measurement
- The importance of the measurement chain

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Literature for Further Reading

- Shock and Vibration Handbook (Harris and Crede, McGraw-Hill 1976)
- Frequency Analysis (Brüel & Kjær Handbook BT 0007-11)
- Brüel & Kjær Technical Review
  - No.1 - 1988 (BV 0033-11)

List of Symbols and Notation

Signal Parameters

t = Time
f = Frequency
ω = Angular frequency
RMS = Root Mean Square

Physical Parameters

d = Displacement
v = Velocity
a = Acceleration
D = Peak Displacement
V = Peak Velocity
A = Peak Acceleration
k = Spring Constant
m, m_1 = Masses
c, c_1 = Damping Coefficients
F = Force

Units

m = Meter
s = Second
Hz = Hertz

dB = Decibel

Electrical Parameters

V = Voltage (Volt)
Q = Charge (Coulomb)
Z = Impedance (Ohm)
R = Resistance (Ohm)
C = Capacitance (Farad)
i = Current (Ampere)
A = Amplification factor (Gain)